SENIOR SIX MID-TERM ONE 2019
APPLIED MATHEMATICS TWO (P425/2)
Time: 3 hours

INSTRUCTIONS:

- *Attempt all questions in section A and five from section B*

- *Any additional question(s) will not be marked.*
- *Begin each question on a fresh sheet of paper.*
- *Mathematical tables with a list of formulae and square papers are provided.*
- *Silent non-programmable science calculators may be used*
- *Indicate all questions attempted in section B on the first page of answer sheets.*

SECTION A (40marks)

1. A particle is projected vertically upwards with a speed of 30ms^{-1} from a point O. Find the time taken before it just approaches point O again.
    (05 marks)

2. Bag P contains 3 green balls and 5 blue balls. Bag Q contains 2 green balls and 4 blue balls. A bag is selected at random and two balls are drawn from it without replacement. Find the probability that;
    (a)   they are both green.
    (b)   they are both green from bag Q.   *(05 marks)*

3. Below are marks of pupils selected in reading and writing for a nursery school:

	A	B	C	D	E	F	G
Writing	63	81	73	23	33	41	53
Reading	77	69	84	61	58	62	69

    Calculate the rank correlation coefficient and comment about its significance at 5% level.   *(05 marks)*

4. Using six(6) ordinates of the trapezium rule, estimate the value of $\int_0^1 e^{(1-x)}dx$ to four significant figures.   *(05 marks)*

5. A constant force $F = (-2i + j)N$ acts on a particle as it moves along a straight wire from A with a position vector $A(2i + j)m$ to point $B(-i + 3j)m$ in 4seconds. Find the average rate at which F is working.
    (05 marks)

6. A random variable has a distribution of form $f(x) = c\left(\frac{4}{5}\right)^x$; $x = 0,1,2. ...$
   Find the expected value of the distribution. *(05 marks)*

7. A particle of weight 16N rests on a horizontal table. The coefficient of friction between the particle and the surface of the table is 0.25. A force P is applied to the particle such that it is just about to slide. Find the value of P given that this force acts upwards at 45^0 to the horizontal. *(05 marks)*

8. Given that the numbers A and B with A=7.35, B = 92.4 measured to the nearest decimal place indicated.
   Determine the:
   (a) absolute error in $\frac{A}{B}$.
   (b) limits within which the quotient $\frac{A}{B}$ lies correct to 3 decimal places. *(05 marks)*

SECTION B (60marks)
Answer only five(5) questions from this section.

9. (a) A particle of weight W is placed on an inclined plane of inclination θ supported by a force, P acting at an angle $\boldsymbol{\alpha}$ to the plane such that it prevents it from sliding downwards. Prove that the minimum force is given by $W sin(\theta - \lambda)$ where λ is the angle of friction. *(06 marks)*

   (b) A particle of mass m is placed on a rough plane inclined at an angle, 30^0 to the horizontal. Given that the angle of friction $\lambda > 30^0$, show that the minimum force required to move the body up the plane is given by $\frac{1}{2}mg\left(cos\lambda + \sqrt{3}sin\lambda\right)$ *(06 marks)*

10. (a) A pump ejects 12000kg of water at a speed of 4ms^{-1} in 40seconds. Find the average rate at which the pump is working. *(06 marks)*

    (b) A pump draws water from a tank and issues it from the end of a horse which is 2.5m vertically above the level from which the water is drawn. The cross-sectional area of the horse is $10cm^2$ and the water leaves the end of the horse at 5ms^{-1}. Find the rate at which the pump is working. (Take density of water $= 1000$kgm^{-3}) *(06 marks)*

11. (a) The forces $\begin{pmatrix} 4 \\ -1 \end{pmatrix}, \begin{pmatrix} -5 \\ 3 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 5 \\ 0 \end{pmatrix} N$ N act at the respective points $(2,2), (2,-2), (-2,2)$ *and* $(-2,-2)$. Find the equation of the line of action the resultant force and also find where it cuts the x-axis.
(06 marks)

(b) The magnitude of the resultant of the forces $6N$ and $12N$ is $6\sqrt{2}N$. Find the
(i) angle between the $6N$ *and* $12N$ forces.
(ii) direction of the resultant force. *(06 marks)*

12. In a particular year 80 children enter a certain large infant's school. The ages of the children on the day they first attend the school are given in the table below:

Age in years							
Number of children	0	10	35	18	10	5	2

(a) Represent the entries on a histogram and use it to estimate the modal age.

(b) Estimate the:
(i) expected age of joining infantry.
(ii) standard deviation.

(c) Find the probability that a child in the catchment area joins school when still under 5years of age.

13. (a) Show that the equation $x^3 + 3x^2 - 1 = 0$ has a real root between $x = -3$ *and* $x = -2$ *(03 marks)*

(b) Using linear interpolation find the first approximation for this root.
(03 marks)

(c) Using Newton-Raphson formula, find the value of the root, correct to 3 significant figures. *(06 marks)*

14. (a) Show the Newton-Raphson formula for finding the reciprocal of a number N is $x_{n+1} = x_n(2 - Nx_n), n = 0, 1, 2, \ldots\ldots$ *(04 marks)*

(b) Draw a flow chart that reads N and x_o the initial approximation, computes and prints the reciprocal of N after four iterations correct to four decimal places. *(04 marks)*

(c) Taking x_o as 0.1 use your flow chart to find the reciprocal of 15 correct to four decimal places. *(04 marks)*

15. A continuous random variable x has the probability density function;

$$f(x) = \begin{cases} 3kx^2 & ; 0 \le x \le 1 \\ k(4-x) & ; 1 \le x \le 2 \\ 2k & ; 2 \le x \le 3 \\ 0 & ; \quad else\ where \end{cases}$$

(a)   Find the value of k *(02 marks)*

(b)   Obtain the cumulative function $F(x)$ *(06 marks)*

(c)   Use your commutative distribution function to find the interquartile range.   *(04 marks)*

16.   (a)   In a certain farm maize is grown in bags of mean weight $40kg$ and standard deviation of $2kg$. Given that the weight is normally distributed, find;   *(04 marks)*

(i)   the probability that the weight of any bag taken at random will be between $41.0kg$ and $42.5kg$.   *(03 marks)*
(ii)   the percentage of bags whose weight exceeds $4kg$.   *(03 marks)*

(b)   The germination of bean seeds is not easy from experience, Mpanga the expert bean grower knows that on average only 40% of the seeds germinate. Six seeds are planted. Determine;
(i)   the probability that only one seed germinates
(ii)   the most likely number of germinating seeds.   *(06 marks)*

END